Method to analyze electromechanical stability of dielectric elastomers

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Subject to an electric voltage, a layer of a dielectric elastomer reduces its thickness, so that the voltage induces a high electric field. The positive feedback may cause the elastomer to thin down drastically, resulting in an electrical breakdown. The authors show that the electromechanical instability occurs when the Hessian of the free-energy function ceases to be positive definite. Their calculation shows that the stability of the actuator is markedly enhanced by prestresses, agreeing with existing experimental observations. © *2007 American Institute of Physics*.
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Dielectric elastomer actuators have been intensely studied in recent years.[1–13] Possible applications include medical devices, energy harvesters, and space robotics.[14–19] Figure 1 illustrates a planar actuator, consisting of a thin layer of dielectric elastomer sandwiched between two compliant electrodes. A battery applies a voltage between the electrodes, and the two weights apply forces in the plane of the actuator. In response to the change in the voltage, the actuator is capable of rapid and large deformation. Such an actuator, however, is susceptible to an electromechanical instability. As the electric field increases, the elastomer thins down, so that the same voltage will induce an even higher electric field. The positive feedback may cause the elastomer to thin down drastically, resulting in an electrical breakdown.

This electromechanical instability has been reviewed recently,[6] and has long been recognized in the electrical power industry as a failure mode of polymer insulators.[20,21] The existing analysis of the instability is based on a heuristic model of Stark and Garton.[20] It has been unclear how such a model may account for more complex materials and loading conditions. This letter formulates a general method to analyze this instability. We will show that the forces due to the weights can markedly enhance the stability of the actuator. This enhancement is known empirically[2,4,6] but has so far not been understood theoretically.

Our analysis is based on a recent formulation of the nonlinear field theory of deformable dielectrics.[22–25] With the reference to Fig. 1, the elastomer has the dimension $L_1L_2L_3$ in the undeformed state. Subject to the electric voltage Φ and mechanical forces P_1 and P_2, the elastomer deforms to a homogeneous state with stretches λ_1, λ_2, and λ_3 as well as gains a magnitude of electric charge Q on either electrodes. The elastomer is taken to be incompressible, so that $\lambda_3=1/(\lambda_1\lambda_2)$.

Define the nominal electric field by the voltage in the deformed state divided by the thickness of the elastomer in the undeformed state, $\tilde{E}=\Phi/L_3$, and define the nominal electric displacement as the charge on an electrode in the deformed state divided by the area of the electrode in the undeformed state, $\tilde{D}=Q/(L_1L_2)$. By contrast, the true electric field is defined as the voltage divided by the thickness of the elastomer in the current state, $E=\Phi/(\lambda_3L_3)$, and the true electric displacement is defined as the charge divided by the area of the electrode in the deformed state, $D=Q/(\lambda_1L_1\lambda_2L_2)$. Denote the nominal stresses by $s_1=P_1/(L_2L_3)$ and $s_2=P_2/(L_1L_3)$.

The elastomer is taken to be an elastic dielectric, with the free-energy function $W(\lambda_1,\lambda_2,\tilde{D})$. The elastomer, the weights, and the battery constitute a thermodynamic system, characterized by three generalized coordinates $\lambda_1,\lambda_2,\tilde{D}$, and three control parameters P_1,P_2,Φ. The free energy of the system is

$$G=L_1L_2L_3W(\lambda_1,\lambda_2,\tilde{D})-P_1\lambda_1L_1-P_2\lambda_2L_2-\Phi Q. \quad (1)$$

When the generalized coordinates vary by small amounts, $\delta\lambda_1,\delta\lambda_2,\delta\tilde{D}$, the free energy of the system varies by

$$\frac{\delta G}{L_1L_2L_3}=\left(\frac{\partial W}{\partial\lambda_1}-s_1\right)\delta\lambda_1+\left(\frac{\partial W}{\partial\lambda_2}-s_2\right)\delta\lambda_2+\left(\frac{\partial W}{\partial\tilde{D}}-\tilde{E}\right)\delta\tilde{D}+\frac{1}{2}\frac{\partial^2W}{\partial\lambda_1^2}\delta\lambda_1^2+\frac{1}{2}\frac{\partial^2W}{\partial\lambda_2^2}\delta\lambda_2^2+\frac{1}{2}\frac{\partial^2W}{\partial\tilde{D}^2}\delta\tilde{D}^2+\frac{\partial^2W}{\partial\lambda_1\partial\lambda_2}\delta\lambda_1\delta\lambda_2+\frac{\partial^2W}{\partial\lambda_1\partial\tilde{D}}\delta\lambda_1\delta\tilde{D}+\frac{\partial^2W}{\partial\lambda_2\partial\tilde{D}}\delta\lambda_2\delta\tilde{D}. \quad (2)$$

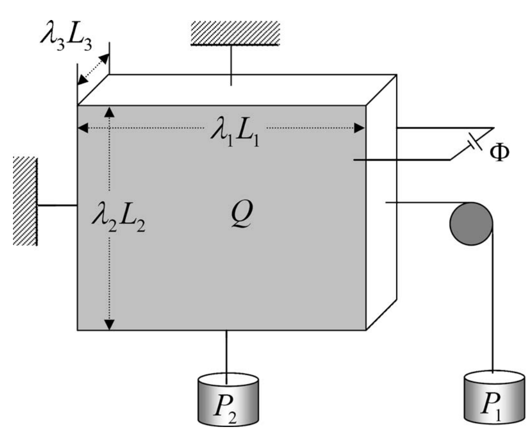

FIG. 1. Layer of a dielectric elastomer coated with two compliant electrodes and loaded by a battery of voltage Φ and by two weights P_1 and P_2. The loads deform the elastomer from lengths L_1, L_2, and L_3 to lengths λ_1L_1, λ_2L_2, and λ_3L_3, as well as induce an electric charge of magnitude Q on either electrode.

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Thermodynamics dictates that a stable equilibrium state should minimize G. In equilibrium, the coefficients of the first-order variations vanish,

$$s_1 = \frac{\partial W}{\partial \lambda_1}, \quad s_2 = \frac{\partial W}{\partial \lambda_2}, \quad \tilde{E} = \frac{\partial W}{\partial \tilde{D}}. \tag{3}$$

To ensure that this equilibrium state minimizes G, the sum of the second-order variations must be positive for arbitrary combination of $\delta\lambda_1, \delta\lambda_2, \delta\tilde{D}$; that is, the Hessian,

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 W}{\partial \lambda_1^2} & \frac{\partial^2 W}{\partial \lambda_1 \partial \lambda_2} & \frac{\partial^2 W}{\partial \lambda_1 \partial \tilde{D}} \\ \frac{\partial^2 W}{\partial \lambda_1 \partial \lambda_2} & \frac{\partial^2 W}{\partial \lambda_2^2} & \frac{\partial^2 W}{\partial \lambda_2 \partial \tilde{D}} \\ \frac{\partial^2 W}{\partial \lambda_1 \partial \tilde{D}} & \frac{\partial^2 W}{\partial \lambda_2 \partial \tilde{D}} & \frac{\partial^2 W}{\partial \tilde{D}^2} \end{bmatrix}, \tag{4}$$

must be positive definite at the equilibrium state.

For a given set of control parameters, P_1, P_2, Φ, Eq. (3) is a set of nonlinear algebraic equations that determine the equilibrium values of the generalized coordinates $\lambda_1, \lambda_2, \tilde{D}$. We now fix the forces P_1 and P_2 but vary the voltage Φ. When the voltage is small, the Hessian is positive definite. When the voltage reaches a critical value Φ^c the Hessian ceases to be positive definite and $\det(\mathbf{H})=0$. The condition $\det(\mathbf{H})=0$, along with the equilibrium equations [Eq. (3)], determine the critical values $\tilde{E}^c$, λ_1^c, λ_2^c, and $\tilde{D}^c$ for any given prestresses s_1 and s_2.

To illustrate the method, consider a model material, called the ideal dielectric elastomer, which has the free-energy function,[23]

$$W(\lambda_1, \lambda_2, \tilde{D}) = \frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_1^{-2}\lambda_2^{-2} - 3) + \frac{\tilde{D}^2}{2\varepsilon}\lambda_1^{-2}\lambda_2^{-2}. \tag{5}$$

The first term is the elastic energy, where μ is the small-strain shear modulus. The second term is the dielectric energy, where ε is the permittivity.

The equilibrium equations [Eq. (3)] become

$$s_1 = \mu(\lambda_1 - \lambda_1^{-3}\lambda_2^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2}, \tag{6a}$$

$$s_2 = \mu(\lambda_2 - \lambda_2^{-3}\lambda_1^{-2}) - \frac{\tilde{D}^2}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2}, \tag{6b}$$

$$\tilde{E} = \frac{\tilde{D}}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2}, \tag{6c}$$

and the Hessian [Eq. (4)] becomes

$$\mathbf{H} = \begin{bmatrix} \mu(1 + 3\lambda_1^{-4}\lambda_2^{-2}) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_1^{-4}\lambda_2^{-2} & 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} \\ 2\mu\lambda_1^{-3}\lambda_2^{-3} + \frac{2\tilde{D}^2}{\varepsilon}\lambda_1^{-3}\lambda_2^{-3} & \mu(1 + 3\lambda_2^{-4}\lambda_1^{-2}) + \frac{3\tilde{D}^2}{\varepsilon}\lambda_2^{-4}\lambda_1^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} \\ -\frac{2\tilde{D}}{\varepsilon}\lambda_1^{-3}\lambda_2^{-2} & -\frac{2\tilde{D}}{\varepsilon}\lambda_2^{-3}\lambda_1^{-2} & \frac{1}{\varepsilon}\lambda_1^{-2}\lambda_2^{-2} \end{bmatrix}. \tag{7}$$

In the special case when the elastomer is under equal biaxial stresses, $s_1=s_2=s$, the stretches are also equal biaxial, $\lambda_1=\lambda_2=\lambda$. The equilibrium condition [Eq. (6)] becomes

$$\frac{\tilde{D}}{\sqrt{\varepsilon\mu}} = \sqrt{\lambda^6 - 1 - \frac{s}{\mu}\lambda^5}, \quad \frac{\tilde{E}}{\sqrt{\mu/\varepsilon}} = \sqrt{\lambda^{-2} - \lambda^{-8} - \frac{s}{\mu}\lambda^{-3}}. \tag{8}$$

For a prescribed mechanical load, s/μ, this pair of equations provides the equilibrium relation between the normalized voltage $\tilde{E}/\sqrt{\mu/\varepsilon}$ and the normalized charge $\tilde{D}/\sqrt{\mu\varepsilon}$, using the stretch λ as a parameter.

Figure 2 shows the effects of the equal biaxial prestress. At a fixed level of the prestress s_1/μ, the function $\tilde{E}(\tilde{D})$ has a peak [Fig. 2(a)]. The left-hand side of each curve corresponds to a positive-definite Hessian, the right-hand side corresponds to a non-positive-definite Hessian, and the peak is determined by $\det(\mathbf{H})=0$. By contrast, the true electric field is a monotonic function of $\tilde{D}$ [Fig. 2(b)]. As the prestress increases, the critical nominal electric field decreases while the critical true electric field increases. The actuation stretch

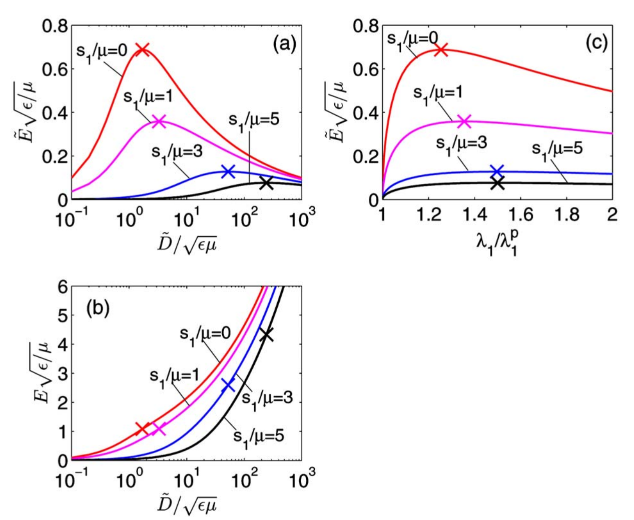

FIG. 2. (Color online) Behavior of a dielectric elastomer actuator under several levels of equal biaxial prestresses: (a) nominal electric field vs nominal electric displacement, (b) true electric field vs nominal electric displacement, and (c) nominal electric field vs actuation stretch. The critical points for instability are marked as crosses.

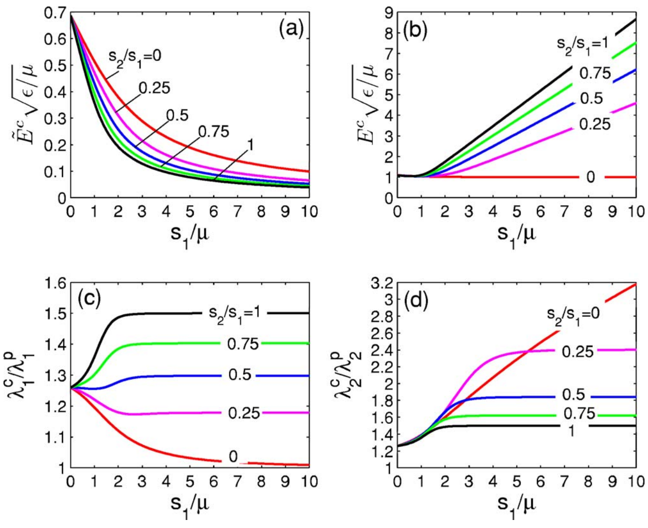

FIG. 3. (Color online) Effects of unequal biaxial prestresses on (a) the critical nominal electric field, (b) the critical true electric field, (c) and (d) and the critical actuation stretches.

is defined as λ_1/λ_1^p, where λ_1^p is the prestretch due to the weights in the absence of the voltage. The biaxial prestress increases the critical actuation stretch [Fig. 2(c)].

In the absence of the prestress, maximizing $\tilde{E}$ in Eq. (8), we obtain the critical stretch $\lambda^c \approx 1.26$, which corresponds to reduction in the thickness by $\sim 37\%$, and is consistent with the maximum thickness strain of $\sim 40\%$ observed experimentally.[1] The critical nominal electric field is $\tilde{E}^c \approx 0.69\sqrt{\mu/\varepsilon}$, which is high when the elastomer is stiff or when the permittivity is low. For representative values $\mu = 10^6$ N/m^2 and $\varepsilon = 4 \times 10^{-11}$ F/m, the critical nominal electric field is $\tilde{E}^c \approx 10^8$ V/m, which is on the same order of magnitude of the reported breakdown fields.[6]

Figure 3 shows the effects of unequal biaxial prestresses, with $s_2/s_1 = 1$ corresponding to equal biaxial prestresses, and $s_2/s_1 = 0$ corresponding to uniaxial prestress. The critical nominal electric field $\tilde{E}^c$ decreases as s_1/μ increases or as s_2/s_1 increases [Fig. 3(a)]. The critical true electric field E^c increases with s_1/μ if $s_2/s_1 > 0$; however, the uniaxial prestretch keeps E^c at an almost constant level as s_1/μ changes. Figures 3(c) and 3(d) show the effects of prestresses on the actuation stretches λ_1^c/λ_1^p and λ_2^c/λ_2^p. It is desirable for an actuator to work under a low voltage and a low true electric field, but generate a high actuation strain. In this connection, note that when the actuator is uniaxially prestressed, the critical true electric field is low, and the actuation stretch in the direction normal to the prestress is large. This trend agrees with the experimental observations.[2,4]

In summary, we have formulated a method to analyze electromechanical stability of dielectric elastomer actuators. While the method is applicable to free-energy function of any form, we have applied the method to the ideal dielectric elastomer. We show that the prestress can markedly increase the actuation stretch. This method can be used to guide the design of actuator configurations, as well as the design of actuator materials.

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